A Crime Rate Forecast and Decomposition Method

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Abstract

Crime forecast is a hotspot in criminology. This paper comes up with a new stochastic crime rate forecast method, namely based on historical age-specific crime rates, we employ SVD (singular value decomposition) in matrix theory to lower the rank of the crime rate matrix, and then transform the time-series vector to a time-series variable problem, then we use time-series analysis to forecast the time-series variable and then the age-specific crime rates. With the forecasted age-specific crime rates and population projection, we obtain the forecasted crude crime rate and then decompose the difference between two crude crime rate into the change in age-specific crime rates, change in age structure and change in sex structure.

Introduction

Criminology is placing increasing emphasis on quantitative methods. Many criminologists are utilizing descriptive statistics and econometric modeling to undertake quantitative crime pattern research. With respect to Criminology's spatial and temporal aspects, the most representative research includes: the hot spot method (Sheman et al., 1989), the relationship between crime and meteorological and temporal factors (Cohn and Rotton, 2000), and crime patterns on vacation (Cohn and Rotton, 2003) etc. In addition, criminologists are still concerned with factors that influence crime rate. They use a plethora of models of differing levels of complexity, from simple correlation to linear modeling to simultaneous equations. They even use modeling to predict crime (Tarling, 1986). This type of research often uses regression techniques, making crime rate a function of economic or population variables (Anderson and Diaz, 1996). Research on crime forecasting has increased as crime prediction has proven itself effective (Clements and Witt, 2005). In 2003, a special section of *International Journal of Forecasting* contained 6 essays and one introduction on crime forecasting (Gorr and Harries, 2003).

Crime forecasting is very important both practically and academically. From a practical standpoint, long-term forecasting can be used to facilitate planning and policy about crimes and prevention. Short-term forecasting can facilitate tactical decisions, such as the deployment of police forces. From a methodological

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perspective, predictive modeling and methods can include exponential smoothing, ARIMA multivariate transfer models, neural networks, and quantitative cointegrated econometric models etc. (Gorr and Harries, 2003). Klepinger and Weis (1985), using a three-step procedure, forecasted property crime and violent crime in the United States. They first divided age-specific crime rates into age, period and cohort effects. Then they utilized regression and transfer functions to analyze period and cohort effects. They were able to create a statistical model to predict the effects of time and formation, enabling them to predict age-specific crime rates.

With respect to crude crime rate change in a population, the changes in age-specific crime rate are only one of many influencing factors contributing to the crude crime rate change. Crude crime rate change over time or the difference between the crude crime rates of two different populations can be broken into three factors: one is a difference in age-specific crime rate, another is difference in age structure, and the last is difference in sex structure. Age is an important determinant on crime. Differing age groups have different crime levels. In many countries, young men are in greater danger of committing crime. A changing proportion of young people within the population has a pronounced effect on crime rate (Schapiro and Ahlberg, 1986). The 15-24 age group commits a large proportion of crime. During the 1960s and early 1970s in the United States, the baby-boomer generation entered this high crime age bracket. Consequently, many researchers have interpreted the rise in crime during this period as the result of the changing age structure (Chilton and Ferdinand, 1970; Spielberger, 1971). Almost all age groups are involved in crimes, and changes in age structure can just account a small portion of the changes in crude crime rates, but such a factor should be taken into account when we analyze the change in crude crime rate (Klepinger and Weis, 1985).

Sex is another important influence on crime. On each individual level, sex has a correlation to crime. In most types of crime, violent crime in particular, male crime rate is higher than that for females. Although research demonstrates the sex structure of a population has very little influence on crime (DeFronzo, 1983), violence is more frequent in populations with a greater proportion of males, causing a negative effect on societal security. An American study in 2004 on the relation of sex and violence indicated societies with a greater proportion of men tend to have a greater crime rates (Hudson and Den Boer, 2004).

Based on previous research, this paper comes up with a new stochastic crime rate forecast method, namely based on historical age-specific crime rates, we employ SVD (singular value decomposition) in matrix theory to lower the rank of the age-specific crime rate matrix, and then transform the time-series vector to a time-series variable problem, then we use time-series analysis to forecast the time-series variable and then the age-specific crime rates. With the forecasted age-specific crime rates and population projection, we obtain the forecasted crude crime rate and then decompose the difference between two crude crime rates into the change in age-specific crime rates, change in age structure and change in sex structure.

Forecasting Age-Specific Crime Rates

The age-specific crime rate forecasting method comes from that in fertility rate and death rate. Demographers have contributed a great deal to the stochastic forecast of fertility and death. Lee and Carter (1992) first apply stochastic method to forecast death rates in the United States. Shen (1995), as well as Li and Shen (1996) use stochastic method to predict China's fertility. They apply singular value decomposition (SVD) on a matrix of historical age-specific fertility rates. Then they use time series forecast method to predict the time series obtained from the SVD results. Based on historical data of age-specific fertility rates, it is assumed that all the determinants of fertility rate change will be reflected in these historical data, as the time series method holds.

SVD of the age-specific crime rate matrix[3]

The singular value decomposition of the matrix is a complete orthogonal decomposition, and is one of the best of many matrix decomposition methods. This method came into wide use at the beginning of the 1970s, with the

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[3]   See Shen(1995) for details

development of computers and information engineering, in the field of matrix rank-reduction and simplification. Its core value is that under the premise that the measureable characteristics of a matrix must not change, it gives the effective ranks of a matrix, and also is the best approximation for rank reduction in terms of the 2-Norm.

C represents a particular type of crime rate, say rape or murder, $C_{0-14}, C_{15-19}, C_{20-24}, \ldots\ldots$ H C_{60-64} H C_{65+} are the crime rates associated with particular age groups. Assuming that there are many age-specific crime data for many years, let C_{tx} denote the crime rate time series where $t = 1, 2, \cdots n$ represents time and could be year or season. $x = 1, 2, \cdots 12$ represents age groups 0-14H15-19H......60-64H65+ respectivelyH Therefore C_{tx} becomes an element in the matrix $A = (C_{tx})_{n \times 12}$. Let g_x be the average of C_{tx}: $g_x = \dfrac{\sum\limits_t C_{tx}}{n}$ H $B = (C_{tx} - g_x)_{n \times 12}$,

Therefore an orthogonal rank reduction exists in $U = [u_1, u_2, \cdots u_{12}] \in R^{n \times 12}$ as well as in $V = [v_1, v_2, \cdots v_{12}] \in R^{12 \times 12}$,

Causing $U^T B V = diag(\sigma_1, \sigma_2, \cdots, \sigma_p] \equiv W$, which is $B = UWV^T$,

Consequently, $B = \sum\limits_{i=1}^{p} U diag(0, \cdots 0, \sigma_i, 0, \cdots 0)V^T$, when $\sigma_i \neq 0$,

$rank(U diag(0, \cdots 0, \sigma_i, 0, \cdots 0)V^T) = 1$, this is to say that inside of $B = \sum\limits_{i=1}^{p} \sigma_i u_i v_i^t$,

$\sigma_1 \geq \sigma_2 \cdots \geq \sigma_p \geq 0, p = \min\{m, n\}$ HThe positive number σ_i is the singular value of rank reduction B, u_i and v_i reflect individually the singular value σ_i left and on the right, the Singular Vector. The singular value and the singular vector have the following relationship:

$$\begin{cases} Bv_i = \sigma_i u_i \\ B^T u_i = \sigma_i v_i \end{cases} \quad (i = 1, 2, \cdots, p)$$

Above is the SVD for matrix B. We must let $m \geq n$, $rank(A) = r \geq s$ (m, n, r, s are all positive integers), let $W_s = diag(\sigma_1, \sigma_2, \cdots, \sigma_s)$ H

$$B_s = U \begin{bmatrix} W_s & 0 \\ 0 & 0 \end{bmatrix} V^T = \sum_{i=1}^{p} \sigma_i u_i v_i^t \text{ H}$$

Then, $rank(A_s) = rank(W_s) = s$, and so $|B - B_s|_F = \min\{|B - C|_F \mid C \in R_s^{m \times n}\}$, $rank(B) = s$.

This explanation falls into the category of the Frobenius norm. B_s is inside of space $R_s^{m \times n}$ ($m \times n$ of term s creates a rank reduction to linear space), and the best approximation to B. In this way, the primary information contained in the crime rate matrix is not lost making it the best approach maintaining fixed rank. For the value B_s, B is the approximate value. Using the $F - $ Norm of $B - B_s$ and the difference of the value approach of $\dfrac{|B - B_s|_F^2}{|B|_F^2}$ and $F - $ Norm of B, $1 - \dfrac{|B - B_s|_F^2}{|B|_F^2}$ shows the descriptiveness (goodness of fit) of B_s.

If $\lambda_i = \dfrac{\sigma_i^2}{\sum_i^{12} \sigma_i^2}$ defined as the ith rate of return of the singular value σ_i, then, satisfying the

condition $\sum_{i=1}^{k} \lambda_i \geq P$ (p is a certain bounded point, e.g.95%)H Matrix $B_{m \times n}$ can use the cross product of the

product of σ_i, u_i and v_i as represented below:

$$B_{m \times n} = \sum_{i=1}^{k} \sigma_i u_i v_i^T .$$

Which is $b_{ij} = \sum_{m=1}^{k} \sigma_i u_{im} v_{jm} \qquad (i = 1, 2, \cdots, n; \quad j = 1, 2, \cdots, 12)$

Time series

After the singular value σ_i contribution rate λ_i are calculated, and satisfy the requirement when $\sum_{i=1}^{k} \lambda_i \geq P$, ($p$ is a certain bounded point, e.g.95%). the rank that needs to make the approximation is determined, say rank 1. Then we achieve $B = (C_{tx} - g_x)_{n \times 12} = \sigma_1 u_1 v_1^T$ HLet $b_x = \sigma_1 v_{x1}$ H $F_t = u_{t1}$, and therefore $C_{tx} = g_x + b_x F_t$. F_t describes the dynamic tendency of crime rate's axial direction. This is because of the sequence the arises in the crime rate matrix and is the object of series modeling research. After determining the time series of F_t, the ARIMA (p, d, q) model is chosen to fit the forecast time series as modeled below:

where,

$\nabla^d = (1\text{-B})^d$, is d^{th} order difference; B is delay factor;

$\Phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p$, is p^{th} order autoregressive multinomial;

$\Theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^p$, is q^{th} order moving average multinomial;

$\{\varepsilon_t\}$ is white noise series, and $E\{\varepsilon_t\} = 0$ $Var(\varepsilon_t) = \sigma_\varepsilon^2$

or using an auto-regressive model to fit the time sequence as shown below:

$x_t = T_t + \varepsilon_t$

$\varepsilon_t = \phi_1 \varepsilon_{t-1} + \phi_2 \varepsilon_{t-2} + \cdots - \phi_p \varepsilon_{t-p} + a_t$

$E(a_t) = 0, \quad Var(a_t) = \sigma^2, \quad Cov(a_t, a_{t-i}) = 0,$

After the time series modeling is determined, the future time series data for F_t can be determined, then bringing the forecast series back into $C_{tx} = g_x + b_x F_t$, the future age-specific crime rate information can be

determined. Of course, if rank 1 can not satisfy the requirement $\sum_{i=1}^{k} \lambda_i \geq P$ (p is a certain bounded point,

e.g.95%) for $B_{m \times n} = \sum_{i=1}^{k} \sigma_i u_i v_i^T$, we have to use 2 or more column vectors, namely u_{t2} or u_{t3}, we just make 2 or more time series forecast.

Crude Crime Rate Forecasting

There are many methods for forecasting crude crime rate. This paper not only seeks to forecast, but also to decompose the difference between two crude crime rates. Therefore, this paper first calculates the age-specific crime rates as mentioned above, then we project future population. With both the forecasted age-specific crime rates and projected population data, we obtain future crude crime rate, and then decompose the difference of two crude crime rates.

Population forecast model

The most commonly-used population projection model is the so called cohort-component method. In this paper, we integrate the cohort-component method with parity-specific SRBs and fertility rates.

$$\begin{cases} x(t+1) = H(t) \cdot x(t) + (\sum_{j=1}^{3} \frac{100}{100 + SRB^j} \bullet \beta^j(t) \cdot B^j(t)) \cdot (x(t) + x^*(t+1))/2 \\ x(0) = x(t_0) \end{cases}$$

Where $x(t) = \begin{bmatrix} x_0(t) \\ x_1(t) \\ \vdots \\ x_M(t) \end{bmatrix}$ $x(t+1) = \begin{bmatrix} x_0(t+1) \\ x_1(t+1) \\ \vdots \\ x_M(t+1) \end{bmatrix}$ $x*(t+1) = \begin{bmatrix} 0 \\ x_1(t+1) \\ \vdots \\ x_M(t+1) \end{bmatrix}$,

where $x_a(t)$ denotes the number of females aged a in the year t, $x_{a+1}(t+1)$ represents the number of female population aged $a+1$ in the year $t+1$. M is the oldest age, and $x(0)$ denotes the population number in the baseline year. $H(t) = \begin{bmatrix} 0 & \cdots & 0 & 0 \\ \eta_0(t) & \cdots & 0 & 0 \\ \ddots & \ddots & \vdots & \vdots \\ 0 & \cdots & \eta_{M-1}(t) & 0 \end{bmatrix}$, where $\eta_a(t)$ is the survival rate for females aged a in the

year t. SRB^j denotes the sex ratio at birth of birth order j, and $\beta^j(t)$ is the total fertility rate for birth order j in the year t. $B^j(t) = \begin{bmatrix} 0 & \cdots & b^j_{15}(t) & \cdots & b^j_{49}(t) & \cdots & 0 \\ \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & 0 \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \end{bmatrix}$, with $b^j_i(t) = \eta_{00}(t)h^j_i(t)$ $i = 15, \cdots 49$, where

$\eta_{00}(t)$ is the female infant survival rate in the year t and $h_i(t)$ represents the fertility pattern for women aged

i in the year t with $h_i(t)$ normalized, so that $\sum\limits_{i=15}^{49} h_i(t) = 1$. Formulas for the simulation of the age structure of males are analogous to these.

Using the above model and baseline data, we can obtain the age and sex structure of population in a given year.

Future Crude Crime Rate

The male age group of a given year is represented by $P_{0-14}^m, P_{15-19}^m, P_{20-24}^m, \ldots\ldots$ Н P_{60-64}^m Н P_{65+}^m Нfemale by $P_{0-14}^f, P_{15-19}^f, P_{20-24}^f, \ldots\ldots$ Н P_{60-64}^f Н P_{65+}^f НThe following crime rate rates correspond to the above male and female population groups, $C_{0-14}^m, C_{15-19}^m, C_{20-24}^m, \ldots\ldots$ Н C_{60-64}^m Н C_{65+}^m and C_{0-14}^f Н C_{15-19}^f Н C_{20-24}^f Н $\ldots\ldots$ Н C_{60-64}^f Н C_{65+}^f

Thus, the male occurrences of a particular type of crime are $\sum\limits_i P_i^m C_i^m$ Нfemale occurrences $\sum\limits_i P_i^f C_i^f$, and the crude crime rate is represented by $CR = \dfrac{\sum\limits_i P_i^m C_i^m + \sum\limits_i P_i^f C_i^f}{\sum\limits_i P_i^m + P_i^f}$

Decomposing Crude Crime Rate

For two different populations, the two different crude crime rates, the difference between two crude crime rates can de decompose into the change in age-specific crime rates, change in age structure and change in sex structure, as follows.

$_1P_i^m$ represents age i males in population 1, the total male population $_1P^m = \sum\limits_i {}_1P_i^m$ Нtotal female population $_1P^f = \sum\limits_i {}_1P_i^f$ Нsex ratio $SR = \dfrac{_1P^m}{_1P^f}$

The crude crime rate is

$\dfrac{\sum\limits_i {}_1P_i^m C_i^m + \sum\limits_i {}_1P_i^f C_i^f}{_1P^m + {}_1P^f} = \dfrac{\sum\limits_i {}_1P_i^m C_i^m + \sum\limits_i {}_1P_i^f C_i^f}{_1P}$ The crime rate in another population, Population 2 is

represented by $\dfrac{\sum\limits_i {}_2P_i^m C_i^m + \sum\limits_i {}_2P_i^f C_i^f}{_2P^m + {}_2P^f} = \dfrac{\sum\limits_i {}_2P_i^m C_i^m + \sum\limits_i {}_2P_i^f C_i^f}{_2P}$ Нso the difference between the crime rates of populations 1 and 2 is

$$\frac{\sum_i {}_2P_i^m {}_2C_i^m + \sum_i {}_2P_i^f {}_2C_i^f}{{}_2P} - \frac{\sum_i {}_1P_i^m C_i^m + \sum_i {}_1P_i^f C_i^f}{{}_1P} =$$

$$\sum_i \frac{({}_2C_i^m - C_i^m) {}_2P_i^m}{{}_2P} + \sum_i \frac{({}_2C_i^f - C_i^f) {}_2P_i^f}{{}_2P} +$$

$$\sum_i \left(\frac{{}_2P_i^m}{{}_2P} - \frac{{}_2P_i^m}{{}_2P^m} \times \frac{SR}{1+SR} \right) \times C_i^m + \sum_i \left(\frac{{}_2P_i^m}{{}_2P^m} \times \frac{SR}{1+SR} - \frac{{}_1P_i^m}{{}_1P} \right) \times C_i^m +$$

$$\sum_i \left(\frac{{}_2P_i^f}{{}_2P} - \frac{{}_2P_i^f}{{}_2P^f} \times \frac{SR}{1+SR} \right) \times C_i^f + \sum_i \left(\frac{{}_2P_i^f}{{}_2P^f} \times \frac{SR}{1+SR} - \frac{{}_1P_i^f}{{}_1P} \right) \times C_i^f$$

where

$\sum_i \frac{({}_2C_i^m - C_i^m) {}_2P_i^m}{{}_2P}$ represents the effect of the difference of age-specific crime rates of males in the two different populations on the difference between the two crude crime rates.

$\sum_i \frac{({}_2C_i^f - C_i^f) {}_2P_i^f}{{}_2P}$ represents the effects of female age-specific crime rate difference on the difference between the two crude crime rates.

$\sum_i \left(\frac{{}_2P_i^m}{{}_2P} - \frac{{}_2P_i^m}{{}_2P^m} \times \frac{SR}{1+SR} \right) \times C_i^m$ represents the effects of male sex structure difference on the difference between the two crude crime rates.

$\sum_i \left(\frac{{}_2P_i^m}{{}_2P^m} \times \frac{SR}{1+SR} - \frac{{}_1P_i^m}{{}_1P} \right) \times C_i^m$ represents the effects of male age structure on the difference between the two crude crime rates.

$\sum_i \left(\frac{{}_2P_i^f}{{}_2P} - \frac{{}_2P_i^f}{{}_2P^f} \times \frac{SR}{1+SR} \right) \times C_i^f$ represents the effects of female sex structure difference on the difference between the two crude crime rates. on the difference in crime rates

$\sum_i \left(\frac{{}_2P_i^f}{{}_2P^f} \times \frac{SR}{1+SR} - \frac{{}_1P_i^f}{{}_1P} \right) \times C_i^f$ represents the effects of female age structure on the difference between the two crude crime rates.

Conclusion

Crime levels are a sensitive political issue, however pertinent governmental departments do not forecast them well. The Home Office in Britain announced their statistical model for its property crime forecast. Their dates ranged from the early 1950s to the late 1990s (Dhiri et al., 1999). But the model does not fit well for the late 1990s data. Using of the 1952 to 1996 annual data relating to unlawful entry for the purpose of theft to predict the 1997 data has shown that the crime rate would drop by 4.3%, the actual crime rate dropped by a staggering 13.6%, even if the given explanatory variables are actual values. In the same way their 1998 prediction was for a decrease of 1.1%, whereas the real decrease was 5.0%. In reality, this crime data does not contain enough

information to make predictions with reasonable precision. In the current intellectual standard, no matter what technique or model is used, there is not likely to be any method of predicting more accurately using this data (Ormerod and Smith, 2010).

This paper provides a method for forecasting age-specific crime rate by age using singular value decomposition and time series modeling. Crime forecasting in its qualitative and quantitative aspects reveals phenomena and dynamic patterns of behavior. Obviously, there are many variables that affect crime rate, for example, income distribution, educational attainment, employment environment, urbanization and poverty (Entorf and Spengler, 2000; Kelly, 2000; Raphael and Winter-Ebmer, 2001; Fajnzylber et al., 2002; Lochner and Moretti, 2004). When using time series to make forecasts, the time series technique dictates all of the information that can be gleaned from the historical data. Time series cannot generally demonstrate a direct relationship between those determinants with crime rate (Anderson et al., 1996).

Using the age-specific crime rate as a foundation, the crude crime rate can be determined through population forecasting. This paper decompose the crude crime rate difference between two populations into three parts: the part that is affected by differences in age-specific crime rates, the part that is affected by differences in age structure, and the part that is affected by differences in sex structure. This can aid in understanding changes in crude crime rate, or in differences in crime rate among different parts of a population. Using this paper's methodology, it is possible to forecast future changes in crime rate, as well as decompose the differences. It is only limited to the availability of data, as this paper only deals with methodology.

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